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REVIEW ARTICLE

GENERAL GRAVITATIONAL INTENSITY (OR ACCELERATION DUE TO GRAVITY) BASED UPON THE GOLDEN METRIC TENSOR IN SPHERICAL POLAR COORDINATES (PAPER II)

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ABSTRACT

In this paper we introduced super general geodesic equation and golden metric tensors. We derived linear velocity vector and linear acceleration vector using golden metric tensor in spherical polar coordinates. Coefficients of affine connection were evaluated based upon golden metric tensor. Based on the evaluated linear velocity vector and acceleration vector, we are in position to obtain super general geodesic planetary equation in terms of r, θ, ϕ and x^0 to obtain Riemannian acceleration due to gravity in terms of r, θ, ϕ and x^0 known as gravitational scalar potential that played an important role in dealing with planetary phenomenon.

Key words: Golden metric tensor, Geodesic equation, coefficient of affine connection.

INTRODUCTION

Riemannian Acceleration Due To Gravity

Riemannian acceleration due to gravity in terms of r, θ, ϕ and x^0 is given by

$$a_H + \frac{1}{(m_I)_H}\left\{\frac{d}{d\tau}[(m_I)_H]\right\}u_H = g_H \quad (1)$$

Where a_H = Riemannian acceleration vector [6]
u_H = Riemannian velocity vector
g_H = Riemannian acceleration due to gravity
$(m_I)_H$ = Riemannian inertial mass

Using

$$u_r = \sqrt{g_{11}}\,r = \left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}} r$$

and

$$a_r = a^1\left(1 + \frac{2}{c^2}f\right)^{-\frac{1}{2}} \quad (2)$$

From equation (60) above we have

$$a_r = \left(1 + \frac{2}{c^2}\right)^{-\frac{1}{2}}\left[r \quad \frac{1}{c^2}\left(1 + \frac{2}{c^2}f\right)^{-1} f_{,1}\,(r)^2 \quad r\left(1 + \frac{2}{c^2}f\right)(\theta)^2 \quad r\sin^2\theta\left(1 + \frac{2}{c^2}f\right)(\phi)^2\right]$$

Hence Riemannian radial acceleration due to gravity is given by

$$a_r + \frac{1}{(m_I)_r}\left\{\frac{d}{d\tau}[(m_I)_r]\right\}u_r = g_r \quad (3)$$

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Putting (60) into equation (3) we have

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[r \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1}\,(r)^2 \quad r\left(1+\frac{2}{c^2}f\right)(\theta)^2 \quad r\sin^2\theta\left(1+\frac{2}{c^2}f\right)(\phi)^2\right]$$

$$+\frac{1}{(m_I)_r}\left\{\frac{d}{d\tau}[(m_I)_r]\right\}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r = g_r \tag{4}$$

Riemannian θ acceleration due to gravity is given by

$$a_\theta+\frac{1}{(m_I)_\theta}\left\{\frac{d}{d\tau}[(m_I)_\theta]\right\}u_\theta = g_\theta \tag{5}$$

Putting equation (61) into equation (5) we get

$$r\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\theta+\frac{2}{r}(r\theta) \quad \sin\theta\cos\theta(\phi)^2\right]$$

$$+\frac{1}{(m_I)_\theta}\left\{\frac{d}{d\tau}[(m_I)_\theta]\right\}\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\theta = g_\theta$$

Riemannian ϕ acceleration due to gravity is given by

$$a_\phi+\frac{1}{(m_I)_\phi}\left\{\frac{d}{d\tau}[(m_I)_\phi]\right\}u_\phi = g_\phi \tag{6}$$

Putting equation (62) into equation (7) we get

$$r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\phi+\frac{2}{r}(r\theta)+2\cot\theta\,(\theta\phi)\right]$$

$$+\frac{1}{(m_I)_\phi}\left\{\frac{d}{d\tau}[(m_I)_\phi]\right\}r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\phi = g_\phi \tag{7}$$

Riemannian x^0 acceleration due to gravity is given by

$$a_0+\frac{1}{(m_I)_0}\left\{\frac{d}{d\tau}[(m_I)_0]\right\}u_0 = g_0 \tag{8}$$

Putting equation (63) into equation (9) we have

$$\left(1+\frac{2}{c^2}f\right)\iota\left[ct+\frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1}f_{,1}\right] \tag{9}$$

$$+\frac{1}{(m_I)_0}\left\{\frac{d}{d\tau}[(m_I)_0]\right\}\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\iota ct = g_0 \tag{10}$$

Hence equation (4), (6), (8) and (10) are super general planetary equations for the components of r, θ, ϕ and x^0.
Using equation (3.2) in paper I, we get

$$(m_I)_H = (m_I)_P = \left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0$$

$$(m_I)_H = \left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0$$

Followed by

$$(m_I)_r = \left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \tag{11}$$

$$(m_I)_\theta = \left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \tag{12}$$

$$(m_I)_\phi = \left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \tag{13}$$

And

$$(m_I)_0 = \left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}f\right)^{\frac{1}{2}} m_0 \qquad (14)$$

Putting equations (11), (12), (13) and (14) into the equation (4), (6), (8) and (10) we get,

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[r \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,1(r)^2 \quad r\left(1+\frac{2}{c^2}f\right)(\theta)^2 \quad r\left(1+\frac{2}{c^2}f\right)\sin^2\theta\left(1+\frac{2}{c^2}f\right)(\phi)^2\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\right\}\right] \equiv g_r \qquad (15)$$

Equation (15) above give rise to super general radial gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.

Hence equation (6) becomes

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\left[\theta+\frac{2}{r}(r\theta) \quad \sin\theta\cos\theta\,(\phi)\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\theta\right\}\right] \equiv g_\theta \qquad (16)$$

Equation (16) above give rise to general θ gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.

Equation (8) becomes

$$\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\sin\theta\left[\phi+\frac{2}{r}(r\phi)+2\cot\theta\,(\theta\phi)\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} r\sin\theta\,\phi\right\}\right] \equiv g_\phi \qquad (17)$$

Equation (17) above give rise to ϕ gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.
Equation (10) reduced to

$$\left(1+\frac{2}{c^2}f\right)\iota\left[ct+\frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1} f_{,1}\right]$$

$$+\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\frac{1}{\left[1-\frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}}\left\{\frac{d}{d\tau}\left[1 \quad \frac{u^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}\right]^{\frac{1}{2}}\left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}} \iota ct\right\}\right] \equiv g_0 \qquad (18)$$

Equation (18) above give rise to x^0 gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor.
Hence equation (15), (16), (17) and (18) are referred to as general gravitational intensity (or acceleration due to gravity) based upon the golden metric tensor in spherical polar coordinates.

In our paper 1 title "General Linear Acceleration Vector Based on The Golden Metric Tensor in Spherical Polar Coordinates". We derived general linear acceleration vector as shown below;

Velocity vector

$$\underline{u}_H = \left(\sqrt{g_{11}}x^1, \sqrt{g_{22}}x^2, \sqrt{g_{33}}x^3, \sqrt{g_{00}}x^0\right) \qquad (18)$$

Where $g_{\mu v}$ are golden metric tensor given as:

$$g_{11} = \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (19a)$$

$$g_{22} = r^2 \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (19b)$$

$$g_{33} = r^2 \sin^2\theta \left(1 + \frac{2}{c^2} f\right)^{-1} \quad (19c)$$

$$g_{00} = \left(1 + \frac{2}{c^2} f\right) \quad (19d)$$

$$g_{uv} = 0, \text{otherwise} \quad (19e)$$

Where f is the gravitational scalar potential of the space time, the golden metric tensor and also contains the following physical effects:

- Gravitational space contraction
- Gravitational time dilation
- Gravitational polar angle contraction
- Gravitational azimuthal angle contraction

Putting equations (19a) to (19d) into equation (19) we get;

$$u_r = \sqrt{g_{11}} r = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} r \quad (20)$$

$$u_\theta = \sqrt{g_{22}} \theta = r \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \theta \quad (21)$$

$$u_\varphi = \sqrt{g_{33}} \varphi = r\sin\theta \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} \varphi \quad (22)$$

$$u_0 = \sqrt{g_{00}} x^0 = \left(1 + \frac{2}{c^2} f\right)^{-\frac{1}{2}} ict \quad (23)$$

Where we have make use of Golden metric tensor in equations (11)-(14)

Acceleration Tensor

Acceleration tensors define by geodesics equation as:

$$a^\mu = x^\mu + \Gamma^\mu_{\alpha\beta} x^\alpha x^\beta \quad (24)$$

Where $\Gamma^\mu_{\alpha\beta}$ is defined as coefficient of affine connection gives as:

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\varepsilon} \left(g_{\alpha\varepsilon,\beta} + g_{\varepsilon\beta,\alpha} \quad g_{\alpha\beta,\varepsilon}\right) \quad (25)$$

Putting $\mu = 1$

$$a^1 = r + \Gamma^1_{11} (r)^2 + \Gamma^1_{22} (\theta)^2 + \Gamma^1_{33} (\phi)^2 \quad (26)$$

Putting $\mu = 2$

$$a^2 = \theta^2 + 2\Gamma^2_{12} (r\theta) + \Gamma^2_{33} (\phi)^2 \quad (27)$$

Putting $\mu = 3$

$$a^3 = \phi^2 + 2\Gamma^3_{13} (r\phi) + 2\Gamma^3_{23} (\theta\phi) \quad (28)$$

Putting $\mu = 0$

$$a^0 = ct + 2c\Gamma^0_{01}(tr)) \tag{29}$$

Using

$$x^0 = ct$$

By employing equation (25) above, the evaluated coefficients of affine connection gives:

$$\Gamma^1_{00} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)f_{,1} \tag{30}$$

$$\Gamma^1_{01} = \Gamma^1_{10} = \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,0} \tag{31}$$

$$\Gamma^1_{11} = \quad \frac{1}{c^2}\left(1+\frac{1}{c^2}f\right)^{-1}f_{,1} \tag{32}$$

$$\Gamma^1_{12} = \Gamma^1_{21} = \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \tag{33}$$

$$\Gamma^1_{13} = \Gamma^1_{31} = \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,3} \tag{34}$$

$$\Gamma^1_{22} = \quad \frac{r^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1} \quad r \tag{35}$$

$$\Gamma^1_{33} = \frac{r^2\sin^2\theta}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1} \quad r\sin^2\theta \tag{36}$$

and

$$\Gamma^2_{00} = \frac{1}{c^2r^2}\left(1+\frac{2}{c^2}f\right)f_{,2} \tag{37}$$

$$\Gamma^2_{02} = \Gamma^2_{20} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,0} \tag{38}$$

$$\Gamma^2_{11} = \frac{1}{c^2r^2}\left(1+\frac{1}{c^2}f\right)^{-1}f_{,2} \tag{39}$$

$$\Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r} \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \tag{40}$$

$$\Gamma^2_{22} = \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \tag{41}$$

$$\Gamma^2_{23} = \Gamma^2_{32} = \quad \frac{2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,3} \tag{42}$$

$$\Gamma^2_{33} = \frac{\sin^2\theta}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,2} \quad \sin\theta\cos\theta \tag{43}$$

and

$$\Gamma^3_{00} = \frac{1}{c^2r^2\sin^2\theta}\left(1+\frac{2}{c^2}f\right)f_{,3} \tag{44}$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,3} \tag{45}$$

$$\Gamma^3_{11} = \frac{1}{c^2r^2\sin^2\theta}\left(1+\frac{2}{c^2}f\right)f_{,3} \tag{46}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1}f_{,1} \tag{47}$$

$$\Gamma^3_{22} = \frac{1}{c^2 \sin^2\theta}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3} \tag{48}$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2} \tag{49}$$

$$\Gamma^3_{33} = \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3} \tag{50}$$

and

$$\Gamma^0_{00} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,0} \tag{51}$$

$$\Gamma^0_{01} = \Gamma^0_{10} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,1} \tag{52}$$

$$\Gamma^0_{02} = \Gamma^0_{20} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,2} \tag{53}$$

$$\Gamma^0_{03} = \Gamma^0_{30} = \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f_{,3} \tag{54}$$

$$\Gamma^0_{01} = \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-3} f_{,0} \tag{55}$$

$$\Gamma^0_{22} = \quad \frac{r^2}{c^2}\left(1+\frac{2}{c^2}f\right)^{-3} f_{,0} \tag{56}$$

$$\Gamma^0_{33} = \quad \frac{r^2\sin^2\theta}{c^2}\left(1+\frac{2}{c^2}f\right)^{-3} f_{,0} \text{ (57)}$$

$$\Gamma^{\mu}_{\alpha\beta} = 0; \quad \text{otherwise} \tag{58}$$

Hence the acceleration vector

$$\underline{a}_H\left(\sqrt{g_{11}}a^1, \sqrt{g_{22}}a^2, \sqrt{g_{33}}a^3, \sqrt{g_{00}}a^0\right) \tag{59}$$

By putting the results of coefficients of affine connection, covariant form of golden metric tensors into equation (59), we obtained acceleration vector equations as:

$$a_r = \sqrt{g_{11}}a^1$$
$$= \left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} a^1$$
$$= \left(1+\frac{2}{c^2}\right)^{-\frac{1}{2}}\left[\begin{matrix} r \quad \frac{1}{c^2}\left(1+\frac{2}{c^2}f\right)^{-1} f,1(r)^2 - r\left(1+\frac{2}{c^2}f\right)(\theta)^2 \\ r\sin^2\theta\left(1+\frac{2}{c^2}f\right)(\phi)^2 \end{matrix}\right] \tag{60}$$

$$a_\theta = \sqrt{g_{22}}a^2$$
$$= r\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}} a^2$$

$$= r\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\theta + \frac{2}{r}(r\theta) \quad \sin\theta\cos\theta(\phi)^2\right] \tag{61}$$

$$a_\phi = \sqrt{g_{33}}a^3$$
$$= r\sin\theta\left(1+\frac{2}{c^2}f\right)^{-\frac{1}{2}}\left[\phi + \frac{2}{r}(r\theta) + 2\cot\theta\,(\theta\phi)\right] \tag{62}$$

$$a_0 = \sqrt{g_{00}}a^0$$
$$= \left(1+\frac{2}{c^2}f\right)\iota\left[ct + \frac{2}{c}\left(1+\frac{2}{c^2}\right)^{-1} f_{,1}\right]$$

Hence equations (60), (61), (62) and (63) are linear acceleration vector equations based upon golden metric tensor.

Conclusion and Results

In our paper I, we have succeeded in obtaining (20) - (23) which referred to as velocity vector equations based upon the golden metric tensor. We further obtained equations (60) – (63) which referred to as linear acceleration vector equations. Also in this paper concerted effort were made in order to obtain equations (15) – (18) which referred to as general gravitational intensity (or acceleration due to gravity) based upon great metric tensor in spherical polar coordinates. .These results obtained in this paper are now available for both physicists and mathematicians alike to apply them in solving planetary problems based upon Riemannian geometry. More effort and time re being devoted in order to offer solution to equations (15) – (18) obtained in this paper with the hope to appear in the next edition of this paper.

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