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RESEARCH ARTICLE

IMPLICATONS ABOUT LATIN SQUARE TRIAL PLAN

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ABSTRACT

In this study, vertical Latin squares, symmetric Latin squares and Latin squares that have been rotated 90 degrees have been examined and the statistical analysis have been carried out through a numerical example. As a result of the analysis, the F test of the Latin square trial and the vertical Latin square trial have been carried out; resulting equal in terms of row, column, trial (Latin) effects sum of squares, the means of squares and trial errors. In the F test of the Latin squares symmetrical to each other, trial (Latin) effects and error (trial error) sum of squares and means of squares remained the same, while the row and column effects sum of squares/means of squares have switched places. In other words, the row effects sum/means of squares of a Latin square resulted equal to the column effects sum/means of squares of the Latin square symmetrical to it. In the F test of the Latin squares which have been rotated 90 degrees; row, column and trial (Latin) effects sum of squares and means of squares and trial errors have resulted the same.

Key Words: Latin Square, Orthogonal, Symmetric.

INTRODUCTION

regarding combination designs and experiments carried out in statistics. A matrix is called Latin square when every element of the set is used only once in each row and each column in a*n x n* sized matrix, defined within a set of n elements. It is called a Latin square because Latin characters were being used when this structure came out for the first time. However, usually $\{1,2,...,n\}$ or $\{0,1,...,n-1\}$ is preferred since it provides convenience in calculations (Seçkin *et al*., 1991). Latin square trial plan requires the number of row, column and treatment to be equal. It's not preferred when the number of treatment is less than 12 (Efe *et al*. 2000).When one or more observations are missing, the missing observations are estimated with appropriate methods. However, if the entire row or column is missing, the analysis of the trial grows difficult.

In Latin square trials, in compliance to the Galois Theory, standard Latin square plans equal to the number of treatment are created. If the number of treatment is t, a total of t!(t-1)!different plans are created from these standard Latin squares and one of them are picked randomly. However, in order to be more practical, first of all, 1 is chosen out of t standard Latin squares. Then, some rows and columns of the chosen standard Latin square are switched randomly and the trial plan obtained is accepted as the incidentally created trial plan (Efe *et al*., 2000).

There are researches carried out regarding the Latin square in various fields. But according to the literature review, no application have been found regarding the comparison of new Latin square trials obtained by symmetrical, vertical and 90 degree rotation, which is one of the existing features of the Latin square, thus, making this study more important.

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The aim of this study is examining the Latin squares which are symmetrical versions of each other and the Latin squares obtained by 90 degree rotation, carrying out the analysis of variance through numerical examples of the trial plans of these Latin squares, and comparing the analysis results.

MATERIALS AND METHODS

Latin square is a design in which the number of rows (r), the number of columns (c) and the number of applications (a) are equal. In every cell of each row and column, there is only one trial unit (Banks, 1965). Since r=c=a, the number of observations is r^2 (Şenoğlu and Acıtaş, 2010). If the material that is going to be used in the experiment differs from two directions, Latin square range is used in order to try the treatments in these units whose differences have been eliminated (Düzgüneş *et al*., 1987). In other words, if the homogeneity of the trial material has been broken by two factors, it is divided into blocks as row and column according to the two factors that broke the homogeneity. The trial error is reduced by creating homogeneity. In this case, Latin Square Trial Plan is applied (Mendeş, 2013).Latin square trial plan has been developed by Fisher in 1925 in order to keep two factors under control while researching the effect of the third factor such as application (Montgomery, 2009). Latin squares might be vertical to each other. Two Latin squares which are vertical to each other may be shown as below (Yazıcı, 2004).

1	2	3	4	5	1	2	3	4	5
2	3	4	5	1	3	4	5	1	2
3	4	5	1	2	5	1	2	3	4
4	5	1	2	3	2	3	4	5	1
5	1	2	3	4	4	5	1	2	3

When demonstrating vertical Latin squares, Latin characters are used for the first square, while Greek characters are used for the second one. That's why vertical Latin squares are called Graeco-Latin squares (Seçkin and friends, 1991).

With this notation, 5x5 vertical squares above and the Graeco-Latin result can be written as below.

A	B	C	D	E	α	β	γ	δ	λ	$A\alpha$	$B\beta$	$C\gamma$	$D\delta$	$E\lambda$
B	C	D	E	A	γ	δ	λ	α	β	$B\gamma$	$C\delta$	$D\lambda$	$E\alpha$	$A\beta$
C	D	E	A	B	λ	α	β	γ	δ	$C\lambda$	$D\alpha$	$E\beta$	$A\gamma$	$B\delta$
D	E	A	B	C	β	γ	δ	λ	α	$D\beta$	$E\gamma$	$A\delta$	$B\lambda$	$C\alpha$
E	A	B	C	D	δ	λ	α	β	γ	$E\delta$	$A\lambda$	$B\alpha$	$C\beta$	$D\gamma$

In Graeco-Latin square design; there are one factor of prime importance and three different justification variables called row, column and Greek characters (Şenoğlu and Acıtaş, 2010). When two Latin squares are combined, no combination of matrices can be present more than once (Lee, 1975). Graeco-Latin square can be defined as the combination of vertical Latin squares (Mandl, 1985; Mead, 1994; Styan *et al*, 2009).

In 1782, Euler has presented a conjecture stating that "k being a non-negative number, there are no vertical Latin square pairs in 4k+2 dimension". The conjecture is true for 2×2 (k = 0).In 1990, G. Tarry has confirmed that there are no vertical Latin square pairs in 6×6 (k = 1). Vertical Latin square pairs in $n\times n$ size are obtained. When n is an odd number, there is always a pair. However, as stated above, if n is an even number such as n = 2, vertical Latin square pairs may not be found. So, for k = 2 and 6, a Graeco-Latin square can't be formed (Kempthorne, 1952).

In 1958, R. C. Bose and Schrikhande have built a Graeco-Latin square of order 22. In 1959, Ernest Tilden Parker has found that Graeco-Latin squares of order 10 can exist. In 1960, Bose, Schrikhande and Parker have proved that apart from 2 and 6 and given that $k\in Z^{+}$, $k\geq 2$, for every n in the dimension of 4k+2, there is at least one vertical Latin square pairs (Yazıcı 2004; Seçkin *et al*, 1991). At most, n-1 vertical Latin squares of order n can be written.

In Latin squares, the rows can be converted into columns and vice versa, meaning that the diagonal symmetry of the square can be written (Nesin and Yazıcı, 2004). Symmetrical Latin squares can be written as below.

1	2	3	4	5	1	4	3	2	5
4	3	1	5	2	2	3	4	5	1
3	4	5	2	1	3	1	5	4	2
2	5	4	1	3	4	5	2	1	3
5	1	2	3	4	5	2	1	3	4

The 90 degree rotation of the square is a Latin transformation. An example is given below. Here, the first and the n'th columns, second and n-1'th columns have been switched and the 90 degree rotation has been obtained (Nesin and Yazıcı, 2004).

1	4	3	2	5	5	2	3	4	1
2	3	4	5	1	1	5	4	3	2
3	1	5	4	2	2	4	5	1	3
4	5	2	1	3	3	1	2	5	4
5	2	1	3	4	4	3	1	2	5

The mathematical model created in order to carry out the statistical analysis of the Latin square trials is stated as below

$$y_{ijk}=\mu+\alpha_i+\beta_j+\gamma_k+\varepsilon_{ijk}, i,j,k=1,2,...,r \qquad (1)$$

In this model,

y_{ijk} : the observation value of row j, column k, trial i,

μ : grand average,

α_i : the effect of trial i,

β_j : the effect of row j,

γ_k : the effect of column k,

ε_{ijk} : random error terms,

(Erbaş and Olmuş, 2006). In model (1), the statistical significance of the effects of trial, row and column are tested. For every situation, the hypotheses are stated below respectively.

$$H_{01}:\alpha_1=\alpha_2=...=\alpha_r=0$$
$$H_{02}:\beta_1=\beta_2=...=\beta_r=0$$
$$H_{03}:\gamma_1=\gamma_2=...=\gamma_r=0$$

The hypotheses are tested using the test statistics obtained by decomposing as grand sum of squares, trial sum of squares, row sum of squares, column sum of squares and error sum of squares. Grand sum of squares are stated as

$$SST=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}(y_{ijk}-\bar{y}...)^2$$

Trial sum of squares, row sum of squares, column sum of squares and error sum of squares are as follows.

$$SST_{trial}=r\sum_{i=1}^{r}(\bar{y}_i..-\bar{y}...)^2$$

$$SSR=r\sum_{j=1}^{r}(\bar{y}_i..-\bar{y}...)^2$$

$$SSC=r\sum_{k=1}^{r}(\bar{y}_{.j.}-\bar{y}...)^2$$

$$SSE=r\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{k=1}^{r}(y_{ijk}-\bar{y}_i..-\bar{y}_{.j.}-\bar{y}.._k+2\bar{y}...)^2$$

To test the hypotheses, test statistics given below are used.

$$F_{trial}=\frac{SST_{trial}/(r-1)}{SSE/(r-1)(r-2)}=\frac{MST}{MSE}$$

$$F_{row}=\frac{SSR/(r-1)}{SSE/(r-1)(r-2)}=\frac{MSR}{MSE}$$

$$F_{column}=\frac{SSC/(r-1)}{SSE/(r-1)(r-2)}=\frac{MSC}{MSE}$$

The ANOVA table for the Latin square trial according to this information has been given in Table 1 (Şenoğlu and Acıtaş, 2010).

Table 1. ANOVA table for Latin square trial

Source	df	Sum of squares	Mean square	F
Treatments	r-1	SST_{trial}	MST	F_{trial}
Rows	r-1	SSR	MSR	F_{row}
Columns	r-1	SSC	MSC	F_{column}
Error	(r-1)(r-2)	SSE	MSE	
General	N-1	SST		

df: Degrees freedom, SST_{trial} : Trial sum of squares, SSR: Row sum of squares, SSC:

Column sum of squares, SSE: Error sum of squares, MST: Trial mean of squares, MSR: Row mean of squares, MSC: Column mean of squares, MSE: Error mean of squares

RESULTS

For the Latin square trial, a Latin square of 5X5 has been formed (Table 2), various situations have been analyzed and the results have been compared. The values used in the study have not been taken from another study, they are values picked randomly for the purpose of application.

Table 2. 5X5 Latin square trial layout

A	100	B	200	C	160	D	80	E	60
B	120	C	160	D	190	E	110	A	150
C	140	D	120	E	190	A	170	B	135
D	225	E	175	A	165	B	125	C	117
E	103	A	124	B	134	C	150	D	130

Table 3. Variance analysis table of the Latin square

Source	Sum of squares	df	Mean square	F	Significant p)
Model	514617.880	13	39585.991	23.708	0.000
Row	5727.440	4	1431.860	0.858	0.516
Column	8275.440	4	2068.860	1.239	0.346
Latin	1331.440	4	332.860	0.199	0.934
Error	20037.120	12	1669.760		
General	534655.000	25			

The F values for row, column and treatment (Latin) effects are 0.858, 1.239 and 0.199 respectively, meanwhile p levels of significance are 0.516>0.01, 0.346>0.01and 0.934>0.01 respectively (Table 3). In this case, statistically there is no significant distinction between the row, column and treatment (Latin) effects. The Latin square vertical to this one is organized as below (Table 4).

Table 4. Vertical 5X5 Latin square trial layout

A	100	B	200	C	160	D	80	E	60
C	140	D	120	E	190	A	170	B	135
E	103	A	124	B	134	C	150	D	130
B	120	C	160	D	190	E	110	A	150
D	225	E	175	A	165	B	125	C	117

The F values for row, column and treatment (Latin) effects are 0.858, 1.239 and 0.199 respectively, meanwhile p levels of significance are 0.516>0.01, 0.346>0.01 and 0.934>0.01 respectively. Statistically there is no significant distinction between the row, column and treatment (Latin) effects. The results obtained from the F test of the vertical Latin squares are the same. So trial error, row, column and Latin square average are equal, also giving the same results in F test and p levels of significance (Table 5).

An example is given below for symmetrical Latin squares (Table 6). The F values for row, column and treatment (Latin) effects are 0.911, 1.504 and 0.212 respectively, meanwhile p levels of significance are 0.488>0.05, 0.262 >0.05 and 0.927>0.05 respectively. Statistically, there is no significant distinction between the row, column and treatment (Latin) effects (Table 7).

Table 5. Variance analysis table of the vertical Latin square

Source	Sum of squares	df	Mean square	F	Significant (p)
Model	514617.880	13	39585.991	23.708	0.000
Row	5727.440	4	1431.860	0.858	0.516
Column	8275.440	4	2068.860	1.239	0.346
Latin	1331.440	4	332.860	0.199	0.934
Error	20037.120	12	1669.760		
General	534655.000	25			

Table 6. 5X5 Latin square trial layout

A	100	B	200	C	160	D	80	E	60
D	190	C	160	A	150	E	110	B	120
C	140	D	120	E	190	B	135	A	170
B	125	E	175	D	225	A	165	C	117
E	103	A	124	B	134	C	150	D	130

Table 7. Variance analysis table of the Latin square

Source	Sum of squares	df	Mean square	F	Significant (p)
Model	515797.880	13	39676.760	25.249	0.000
Row	5727.440	4	1431.860	0.911	0.488
Column	9455.440	4	2363.860	1.504	0.262
Latin	1331.440	4	332.860	0.212	0.927
Error	18857.120	12	1571.427		
General	534655.000	25			

Table 8. 5X5 Symmetrical Latin square trial layout

A	100	D	190	C	140	B	125	E	103
B	200	C	160	D	120	E	175	A	124
C	160	A	150	E	190	D	225	B	134
D	80	E	110	B	135	A	165	C	150
E	60	B	120	A	170	C	117	D	130

In the symmetrical Latin square trial (Table 8); the F values for row, column and treatment (Latin) effects are 1.504, 0.911 and 0.212 respectively, meanwhile p levels of significance are 0.262>0.01, 0.488>0.01 and 0.927>0.01 respectively (Table 9). Statistically, there is no significant distinction between the row, column and treatment (Latin) effects.

Table 9. Variance analysis table of the symmetrical Latin square

Source	Sum of squares	df	Mean square	F	Significant (p)
Model	515797.880	13	39676.298	25.249	0.000
Row	9455.440	4	2363.860	1.504	0.262
Column	5727.440	4	1431.860	0.911	0.488
Latin	1331.440	4	332.860	0.212	0.927
Error	18857.120	12	1571.427		
General	534655.000	25			

Table 10. 5X5 Latin square trial layout

A	100	D	80	C	160	B	200	E	60
B	120	C	160	D	190	E	110	A	150
C	140	A	170	E	190	D	120	B	135
D	225	E	175	B	125	A	165	C	117
E	103	B	134	A	124	C	150	D	130

Table 11. Variance analysis table of the Latin square

Source	Sum of squares	df	Mean square	F	Significant (p)
Model	510721.880	13	39286.298	19.698	0.000
Row	5727.440	4	1431.860	0.718	0.596
Column	4379.440	4	1094.860	0.549	0.703
Latin	1331.440	4	332.860	0.167	0.951
Error	23933.120	12	1994.427		
General	534655.000	25			

Table 12. 5X5 Latin square trial layout after 90 degree rotation

E	60	B	200	C	160	D	80	A	100
A	150	E	110	D	190	C	160	B	120
B	135	D	120	E	190	A	170	C	140
C	117	A	165	B	125	E	175	D	225
D	130	C	150	A	124	B	134	E	103

In symmetrical Latin square trial, trial (Latin) effect and error sum and means of squares did not change, where the row and column means of square have switched. So, the row means of square symmetry in one Latin square trial is equal to the column means of square in the other Latin square trial. Similarly, the column means of square in one symmetrical Latin square is equal to the row means of square in the other one. The results from the F test are the same. So the trial error and the Latin square mean are equal, giving the same results in F test and p levels of significance.

An example is given below regarding the Latin squares after 90 degrees of rotation (Table 10 and Table 11). The F values for row, column and treatment (Latin) effects are 0.718, 0.549 and 0.167 respectively, meanwhile p levels of significance are 0.596>0.05, 0.703>0.05 and 0.951>0.05respectively. Statistically, there is no significant distinction between the row, column and treatment (Latin) effects.

Is obtained as above (Table 12). The test results for the Latin square design organized according to this is as follows.

Table 13. Variance analysis table of the Latin square after 90 degree rotation

Source	Sum of squares	df	Mean square	F	Significant (p)
Model	510721.880	13	39286.298	19.698	0.000
Row	5727.440	4	1431.860	0.718	0.596
Column	4379.440	4	1094.860	0.549	0.703
Latin	1331.440	4	332.860	0.167	0.951
Error	23933.120	12	1994.427		
General	534655.000	25			

In the Latin square trial after 90 degree rotation; the F values for row, column and treatment (Latin) effects are 0.718, 0.549 and 0.167 respectively, meanwhile p levels of significance are 0.596>0.05, 0.703 >0.05 and 0.951>0.05 respectively (Table 13). Statistically, there is no significant distinction between the row, column and treatment (Latin) effects.

When the F test is applied to Latin square trial after 90 degree rotation and the other Latin square trials; trial errors, meaning error means of squares, row and column means of squares are equal. Hence the statistical values of the F test and p levels of significance are equal.

Conclusion

In this study, brief information about Latin square trial has been given and various features have been examined. Vertical Latin squares, symmetrical Latin squares and Latin squares after 90 degree rotation have been formed. In 5X5 Latin square trial, analyses have been carried out using the same values.

The F test carried out after working on vertical Latin squares has given the same results. Row, column and trial (Latin) effects sum and means of squares are equal. So statistically, no significant distinction between the row, column and treatment (Latin) effects have been found. There hasn't been any change in trial errors.

With the F test carried out after working on symmetrical Latin squares, the row sum and means of squares in one of the Latin squares turned out to be equal to the column sum and means of squares in the Latin square trial symmetrical to the other one. The sum and means of trial and error effects have turned out to be the same. However, no statistical difference has been found between row, column and treatment (Latin) effects. There hasn't been any change in trial errors.

The F test applied on Latin squares after 90 degree rotation has given the same results. The sum and means of trial and error effects are the same. So statistically, no significant difference has been found between row, column and treatment (Latin) effects. There hasn't been any change in trial errors.

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