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RESEARCH ARTICLE

SOLVING LINEAR POTENTIAL USING THE VARIATION METHOD

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ABSTRACT

We examine the variation of 1-dimentional quantum mechanics system in linear potential (constant force), using variation wave functions for the ground state up to the second order shift. This approach use simple wave functions to describe the exact Airy solution. The exact solution is compare with solutions obtain from the variation function.

Key Words: Linear Potential, Airy Function, Variation method.

INTRODUCTION

Quantum mechanics was prompted due to failure of classical in explaining a number of microphysical phenomena (Zettilli, 2001). Simple quantum mechanics systems are very useful in education because they introduce important principles of quantum mechanics to the study of hydrogen atom. In quantum mechanics the analysis of various potential is important. In this work we consider a 1-dimensional model of quantum mechanical system describe by linear potential or constant force, that describe the motion of particle falling under gravity the so called 'quantum bouncer' (define by $v(z) = fz \; for \; z \geq 0 \; and \; v(z) = \infty \; for \; z < 0$) (Gibbs 1975). The force in this case is $-mg$, so potential above the ground is mgz. We solve the 1D Schrödinger by the airy function properties (Vallēe et. al. 2004), which can also be analyzed using variation method or other approximation methods to compare with the exact result from the standard methods.

The simplest model problem is provided by a particle in a one-dimensional box (PIB) of length L, in which the particle is constrained to remain between two infinite potential walls. Inside the box the potential is constant and usually taken as the zero of energy. The exact solutions of this problem are known, and have been extensively investigated (Casauban *et al.,* 2007) The linear potential plus the infinite wall where electron is trapped between the surface and weak magnetic force is used to analyze surface –landau-level resonance data giving by $F = ev_f$ H/C. The motion of the bouncer potential also includes system of atoms (Robinett 2009). The variation method is one of two common methods used in quantum mechanics. Variation method does not require knowledge of simpler Hamiltonian that can be solving exactly (Zetilli 2001).

$$-\frac{1}{2}\frac{d^2}{dy^2}\psi(y) + kx\psi(y) = E\psi(y) \qquad 1$$

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We begin by reviewing the solution for a quantum linear potential, in this paper we set the atomic units $h = c = m = 1$ and solve the Schrödinger equation. Changing the variable to $x = \rho y + \sigma$ and defining the parameter

$$\rho = (\frac{\hbar^2}{2mf})^{1/3} \text{ And } \sigma = \frac{E}{F} \qquad 2$$

Simplifying Eqn. (1) and find the result.

$$\frac{d^2\psi_n(y)}{dy^2} = y\psi_n(y) \qquad 3$$

The general solution of Eqn. (3) Are two linearly independent Airy functions $Ai(x - \sigma_n)$ and $Bi(x - \sigma_n)$ (Robinett 2009), the first point to note is that as $\sigma \to \infty$. The $Ai(\sigma) = 0$ as $\lim Bi(\sigma) = \infty$, so the $Bi(\sigma)$ function has an appropriate asymptotic form. The energy Eigenvalue is determine by the condition $\psi_n(y \to \infty) = 0$ and so excluded. Thus are determined by boundary condition imposed by the infinite wall at the origin. Namely that $\psi(y = 0) = Ai(-\sigma_n)$. the quantized energies are then given in terms of the zeros of the well behaved Airy function, $Ai(-\sigma_n)$ with $E_n = +\sigma_n \varepsilon_o$, due to matching of solution at the boundary between two region where the solutions are defined differently ,namely at the origin.

If we label the zeros o f Ai'(y) and Ai(y) functions, $-y_i^{(+)} \; odd \; and \; -y_i^{(-)} \; even$ respectively, the condition below determine the quantized energy eigenvalue since

$$E_I^{(\pm)} = y_i^{(\pm)}(\frac{\hbar^2 F^2}{2m})^{1/3} \qquad 4$$

The first three roots of the Airy function are

$$\sigma_1 = 2.338107 \,, \quad \sigma_2 = 4.087949 \; and \; \sigma_3 = 5.520560$$

(Abramowitz *et al.,* 1972). The exact solutions for the first three states using mathematics for the computations by setting k=1 is

$E_1^{exact} = 1.855759, \quad E_2^{exact} = 3.244609 \; and \; E_2^{exact} = 4.381673$

(Pengpan, 2008). The eigenfunctions Ai are shifted in each case so that have zero at $z = 0$ and by $z < 0$ round down.

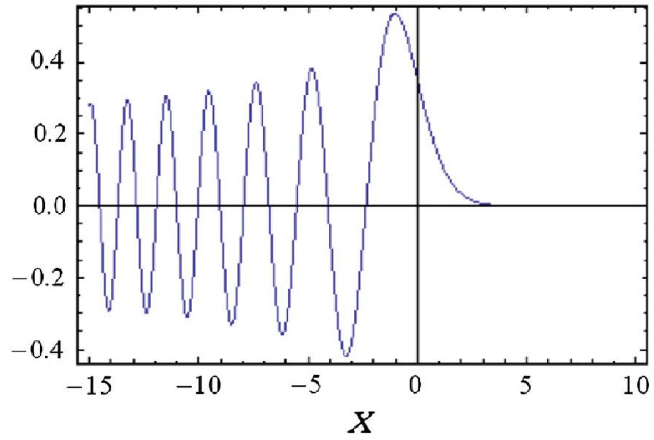

Fig. 1. plot of eigenfunction $Ai(x)$

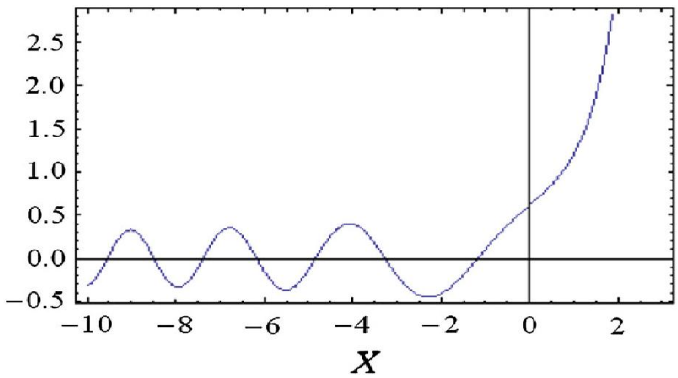

Fig. 2. plot of eigenfunction $Bi(x)$

Winter Variation Approach

The winter variation approach (Winter 1986). is based on the trial wave function which is normalized, if we consider

$$\psi(z) = 2\beta^{3/2} Z e^{(-\beta z)} \quad 5$$

From the variation method,

$$H\Psi_n = E_n \Psi_n \quad 6$$

$$E(\psi) = \frac{\langle\psi|\hat{H}|\psi\rangle}{\langle\psi|\psi\rangle} \quad 7$$

Solving the energy derivative we get

$$\frac{d}{d\beta}\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} = 0 \quad 8$$

$$\delta E(\psi) = 0 \quad 9$$

The result obtain by winter is

$\beta = ({}^{3}/_{2})^{1/3} \; and \; E_1(w) = 1.96553.$

which has a percentage error of 5.6% higher than the exact value of the ground state energy ($E_1^{exact} = 1.85575$).

Variation Wave Function Improvement

The trial wave function is improve by examining the acts of the Airy function solution, for large as $y \; as \; y \rightarrow +\infty$ the Airy function can be in form as (Winter 1986).

$$Ai(z) \approx \frac{1}{2\sqrt{\pi} z^{1/4}} \exp\left(-{}^{2}/_{3} z^{{}^{3}/_{2}}\right) \quad 10$$

Taking the un normalized variation wave function on the asymptotic behavior basis, the wave function will in the form

$$\Psi(z) = z exp(-p z^{{}^{3}/_{2}}) \quad 11$$

The value of p is obtain by the method of trial and error, which is obtain by lowering the energy which gives a value of $p = 0.667$. Using p the ground state energy is found to be

$E_1^{var_1} = 1.861$

which gives an error of 0,28%, also further improving the ground state energy is by replacing the ${}^{3}/_{2}$ in Eqn. 11 with a parameter q which gives a new trial wave function with the form

$$\Psi(z) = z \exp(-p z^q) \quad 12$$

The optimum values of the variation parameters by minimizing the energy are $p = 0.5245 \; and \; q = 1.750$. This parameters leads to an improved value of the energy $E_1^{var_2} = 1.85575$ which is 0.008% higher than the exact energy in the ground state ($E_1^{exact} = 1.85575$). To optimize the energy for the first excited state, the trial wave function will be in form as

$$\Psi(z) = (Pz^2 + Qz)\exp(-p z^q) \quad 13$$

In Eqn.13, Using $p = 0.5245 \; and \; q = 1.750$, and Eqn.13 remain orthogonal with Eqn.12 by taking

${}^{Q}/_{P} = -1.23703$. The value of first excited state energy is $E_2^{var} = 3.5013$. This energy has 7.3% of error higher than exact energy ($E_2^{exact} = 3.24457$).

DISCUSSION

The improvement in this work shows that it is always more difficult enhance the energy of the excited states than the ground state energy, by the variation method and have almost equal result with the exact solution. The variation method gives almost exact solution of the ground state energy than perturbation method, so variation method is mostly use in evaluating the ground state energy.

Conclusion

In this work we show that using a simple wave function to describe the exact Airy function solution for the linear potential problem of quantum mechanics, one can find the ground state and first excited state energies as 0.008% and 7.3% higher than the exact energy values. The ground state energy found by the use of wave function is accurate to the exact value than that reportedly found previously by winter (1986).

REFERENCES

Nouredine, zettilli. Quantum mechanics: concept and application 1. 2001, 488.

Gibbs, R.L. 1975: The quantum bouncer. *Am. J: Phys* 43.25-28.

Vallēe, O. and soares, m. 2014 Airy functions and application to physics (New Jersey: world scientific).

Casauban, J. I., J. P. 2007. Cosentino and A. H. Buep.: *Turk J Phys* 117-121 31.

Robinett, R. W. arXiv: 0909.2209v1 [quanta-ph] 11 Sep 2009

Teparksorn, Pengpan.: arXiv:0812.1069v1 [physics.com-ph] 5 Dec 2008

Abramowitz, M. and I.A. 1972. Stegun, Handbook of mathematical functions (Dover,)

Winter, R.G. quantum physics, (faculty publishing inc.: Davis California, 1986) p.92 and p.333.

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